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The Best Gain-Loss Ratio is a Poor Performance Measure*

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Abstract. The gain-loss ratio is known to enjoy very good properties from a normative point of view. As a confirmation, we show that the best market gain-loss ratio in the presence of a random endowment is an acceptability index, and we provide its dual representation for general probability spaces. However, the gain-loss ratio was designed for finite Ω and works best in that case. For general Ω and in most continuous time models, the best gain-loss is either infinite or fails to be attained. In addition, it displays an odd behavior due to the scale invariance property, which does not seem desirable in this context. Such weaknesses definitely prove that the (best) gain-loss is a *poor* performance measure.

Key words. gain-loss ratio, acceptability indexes, incomplete markets, martingales, quasi-concave optimization, duality methods, market modified risk measures

AMS subject classifications. 46N10, 91G99, 60H99

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1. Introduction.

The gain-loss ratio was introduced by Bernardo and Ledoit [2] to provide an alternative to the classic Sharpe ratio (SR) in portfolio performance evaluation. Cochrane and Saa-Requejo [9] call portfolios with high SR "good deals." These opportunities should, informally speaking, be regarded as quasi arbitrages and therefore should be ruled out. Ruling out good deals, or equivalently restricting SR, produces in turn restrictions on pricing kernels. Restricted pricing kernels are desirable since they provide narrower lower and upper price intervals for contingent claims in comparison to arbitrage free price intervals. This criterion is based on the assumption that a high SR is attractive and a low SR is not. The SR criterion works well in a Gaussian returns context, but in general it does not since it is incompatible with no-arbitrage. In fact a positive gain with finite first moment but infinite variance has zero SR, but it is very attractive as it is an arbitrage. The SR has another drawback: it is not monotone and thus violates a basic axiom in theory of choice. To remedy the aforementioned shortcomings of the SR, Bernardo and Ledoit proposed as performance measure the gain-loss ratio

$$\alpha(X) = \frac{E[X^+]}{E[X^-]},$$

where the expectation is taken under the historical probability measure P. The gain-loss ratio α is well defined on nonnull payoffs X as soon as X^+ or X^- are integrable; it has an intuitive significance and is easy to compute. It also enjoys many properties: monotonicity across Xs;

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