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Mean-Variance-Skewness Portfolio Performance Gauging: A General Shortage Function and Dual Approach

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This paper proposes a nonparametric efficiency measurement approach for the static portfolio selection problem in mean-variance-skewness space. A shortage function is defined that looks for possible increases in return and skewness and decreases in variance. Global optimality is guaranteed for the resulting optimal portfolios. We also establish a link to a proper indirect mean-variance-skewness utility function. For computational reasons, the optimal portfolios resulting from this dual approach are only locally optimal. This framework permits to differentiate between portfolio efficiency and allocative efficiency, and a convexity efficiency component related to the difference between the primal, nonconvex approach and the dual, convex approach. Furthermore, in principle, information can be retrieved about the revealed risk aversion and prudence of investors. An empirical section on a small sample of assets serves as an illustration.

Key words: shortage function; efficient frontier; mean-variance-skewness portfolios; risk aversion; prudence
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1. Introduction

The seminal work of Markowitz (1952) in modern portfolio theory trades off the risk and expected return of a portfolio in a static context. Portfolios whose expected return cannot increase unless their risk increases define an efficient frontier, i.e., a Pareto-optimal subset of portfolios. His work maintains strong assumptions on probability distributions and Von Neumann-Morgenstern utility functions. Furthermore, the computational cost of solving quadratic programs in these days led Sharpe (1963) to propose a simpler "diagonal" model and inspired Sharpe (1964) and Lintner (1965) to develop the capital asset pricing model (CAPM), an equilibrium model assuming that all agents have similar expectations about the market. Widespread tools for gauging portfolio efficiency, such as Sharpe (1966) and Treynor (1965) ratios and Jensen (1968) alpha, have mainly been developed with reference to these developments, and in particular, CAPM. Despite these and later enhancements, the Markowitz model still offers the most general framework.

The main theoretical difficulty with the so-called parametric approach where utility depends on the first and second moments (i.e., mean and variance) of the random variable's distribution is that it is only consistent with expected utility and its von Neumann-Morgenstern axioms of choice when (i) asset processes are normally distributed (hence, higher moments can be ignored), or (ii) investors have quadratic utility functions (e.g., Samuelson 1967). However, a plethora of empirical studies shows that portfolio returns are generally not normally distributed. Furthermore, investors prefer positive skewness because it implies a low probability of obtaining a large negative return. In particular, the observation that increased diversification leads to skewness loss and the widespread phenomenon of imperfectly diversified portfolios may well reveal a preference for positive skewness among investors, rather than simply capital market imperfections (Kraus and Litzenberger 1976, Simkowitz and Beedles 1978, Kane 1982). Theoretically, positive skewness preference is related to the positivity of the third derivative of the utility function: the prudence notion is to marginal utility what risk aversion is to utility.[1] Furthermore, ever

[1] As Kimball (1990, p. 54) states: "The term 'prudence' is meant to suggest the propensity to prepare and forearm oneself in the face of uncertainty, in contrast to 'risk aversion,' which is how much