



# Trimmed L-moments

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## Abstract

Classical estimation methods (least squares, the method of moments and maximum likelihood) work well in regular cases such as the exponential family, but outliers can have undue influence on these methods. We define population *trimmed L-moments* (TL-moments) and corresponding sample TL-moments as robust generalisations of population and sample L-moments. TL-moments assign zero weight to extreme observations, they are easy to compute, their sample variances and covariances can be obtained in closed form, and they are more robust than L-moments are to the presence of outliers. Moreover, a population TL-moment may be well defined where the corresponding population L-moment does not exist: for example, the first population TL-moment is well defined for a Cauchy distribution, but the first population L-moment, the population mean, does not exist. The sample TL-mean is compared with other robust estimators of location.

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## 1. Introduction

Outliers can have undue influence on standard estimation methods such as least squares, the method of moments and maximum likelihood; for example, each of these three methods estimates the mean of a normal population by the sample mean  $\bar{X}$  which is the unique minimum variance unbiased estimator but is not robust to outliers or departures from normality. Thus, if there is concern about extreme observations having undue influence, a robust method of estimation, developed to reduce influence

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