

A Sharper Ratio: A General Measure for Correctly Ranking Non-Normal Investment Risks

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This Version: February 3, 2014

Abstract

While the Sharpe ratio is still the dominant measure for ranking risky investments, much effort has been made over the past three decades to find more robust measures that accommodate non-Normal risks (e.g., “fat tails”). But these measures have failed to map to the actual investor problem except under strong restrictions; numerous ad-hoc measures have arisen to fill the void. We derive a generalized ranking measure that correctly ranks risks relative to the original investor problem for a broad utility-and-probability space. Like the Sharpe ratio, the generalized measure maintains wealth separation for the broad HARA utility class. The generalized measure can also correctly rank risks following different probability distributions, making it a foundation for multi-asset class optimization. This paper also explores the theoretical foundations of risk ranking, including proving a key impossibility theorem: any ranking measure that is valid for non-Normal distributions cannot generically be free from investor preferences. Finally, we show that approximation measures, which have sometimes been used in the past, fail to closely approximate the generalized ratio, even if those approximations are extended to an infinite number of higher moments.

Keywords: Sharpe Ratio, portfolio ranking, infinitely divisible distributions, generalized ranking measure, Maclaurin expansions

JEL Code: G11

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