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# Portfolio performance evaluation with generalized Sharpe ratios: Beyond the mean and variance

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ABSTRACT

This paper presents a theoretically sound portfolio performance measure that takes into account higher moments of distribution. This measure is motivated by a study of the investor's preferences to higher moments of distribution within Expected Utility Theory and an approximation analysis of the optimal capital allocation problem. We show that this performance measure justifies the notion of the Generalized Sharpe Ratio (GSR) introduced by Hodges (1998). We present two methods of practical estimation of the GSR: nonparametric and parametric. For the implementation of the parametric method we derive a closed-form solution for the GSR where the higher moments are calibrated to the normal inverse Gaussian distribution. We illustrate how the GSR can mitigate the shortcomings of the Sharpe ratio in resolution of Sharpe ratio paradoxes and reveal the real performance of portfolios with manipulated Sharpe ratios. We also demonstrate the use of this measure in the performance evaluation of hedge funds.



## 1. Introduction

The Sharpe ratio is a commonly used measure of portfolio performance. However, because it is based on the mean-variance theory, it is valid only for either normally distributed returns or quadratic preferences. In other words, the Sharpe ratio is a meaningful measure of portfolio performance when the risk can be adequately measured by standard deviation. When return distributions are non-normal, the Sharpe ratio can lead to misleading conclusions and unsatisfactory paradoxes, see, for example, Hodges (1998) and Bernardo and Ledoit (2000). For instance, it is well-known that the distribution of hedge fund returns deviates significantly from normality (see, for example, Brooks and Kat, 2002; Agarwal and Naik, 2004; Malkiel and Saha, 2005). Therefore, performance evaluation of hedge funds using the Sharpe ratio seems to be dubious. Moreover, recently a number of papers have shown that the Sharpe ratio is prone to manipulation (see, for example, Leland, 1999; Spurgin, 2001; Goetzmann et al., 2002; Ingersoll et al., 2007). Manipulation of the Sharpe ratio consists largely in selling the upside return potential, thus creating a distribution with high left-tail risk.

The literature on performance evaluation that takes into account higher moments of distribution is a vast one. Motivated by a common interpretation of the Sharpe ratio as a *reward-to-risk* ratio, many researchers replace the standard deviation in the Sharpe ratio by an alternative risk measure. For example, Sortino and Price (1994) replace standard deviation by downside deviation. The examples of the use of the risk measures on the basis of value-at-risk (VaR) include: Dowd (2000) (standard VaR), Favre and Galeano (2002) (modified for skewness and kurtosis VaR), and Rachev et al. (2007) (conditional VaR). Other researchers replace both the risk premium and the standard deviation in the Sharpe ratio with alternative measures of reward and risk. Some examples of this approach are: Stutzer (2000) introduced the Stutzer index which is based on the behavioral hypothesis that investors aim to minimize the probability of returns being below a given threshold. The Omega ratio was introduced by Shadwick and Keating (2002). This measure is expressed as the ratio of the gains with respect to some threshold to the loss with respect to the same threshold. Kaplan and Knowles (2004) introduced the Kappa measure which generalizes the Sortino and Omega ratios. The list can be made much longer and we apologize that not all alternative performance measures[1] can be mentioned here. However, whereas the Sharpe ratio is, in principle, based on Expected Utility Theory which is the cornerstone of modern finance, most of the alternative

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[1] For a brief review of different portfolio performance measures, as well as different reward and risk measures used in performance evaluation, the interested reader can consult Eling and Schuhmacher (2007) and especially Farinelli et al. (2008).

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